

## CLAIMS

What is claimed is:

- 1           1.       A method comprising:  
2           generating a first covariance matrix from a desired mean vector and a desired  
3 covariance matrix of a Bernoulli distribution;  
4           constructing a normal vector using the desired mean vector and the first covariance  
5 matrix; and  
6           generating a sampling vector using the normal vector and a threshold vector, the  
7 sampling vector having the desired mean vector and the desired covariance matrix.
  
- 1           2.       The method of claim 1 wherein generating the first covariance matrix  
2 comprises:  
3           generating an integral expression  $F$  for a first non-diagonal element  $s_{ij}$  of the first  
4 covariance matrix at a row index  $i$  and a column index  $j$ , the integral expression having an  
5 integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$  in the threshold vector at the vector  
6 indices  $i$  and  $j$ ; and  
7           obtaining the first non-diagonal element  $s_{ij}$  using the integral expression  $F$ , a mean  
8  $\mu_k$  of the desired mean vector, and a desired non-diagonal element  $\Sigma_{ij}$  of the desired  
9 covariance matrix.
  
- 1           3.       The method of claim 2 further comprising:  
2           obtaining a diagonal element  $s_{jj}$  of the first covariance matrix at a first row index  $j$   
3 and a first column index  $j$  using the mean  $\mu_j$  at the vector index  $j$ , the diagonal element  
4 being equal to a desired diagonal element  $\Sigma_{jj}$  of the desired covariance matrix.
  
- 1           4.       The method of claim 3 further comprising:  
2           generating a threshold element  $\tau_j$  of the threshold vector at a vector index  $j$ , the  
3 threshold element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1-2\mu_j)$  wherein  $\mu_j$  and  $\sigma_j$  are desired  
4 mean and variance, respectively, at the vector index  $j$  and  $\operatorname{inverf}$  is an inverse error  
5 function.

1           5.       The method of claim 2 wherein constructing the normal vector comprises:  
2           generating normal elements of the normal vector using the desired mean vector and  
3           the first covariance matrix.

1           6.       The method of claim 5 wherein generating the sampling vector comprises:  
2           comparing a normal element  $Y_k$  of the normal vector at a vector index  $k$  with a  
3           corresponding threshold element  $\tau_k$  of the threshold vector at the vector index  $k$ ;  
4           setting a sampling element of the sampling vector at the vector index  $k$  to a first  
5           value if the normal element  $Y_k$  is greater than the corresponding threshold element  $\tau_k$ ; and  
6           setting the sampling element of the sampling vector at the vector index  $k$  to a  
7           second value if the normal element  $Y_k$  is equal to or less than the corresponding threshold  
8           element  $\tau_k$ .

1           7.       The method of claim 2 wherein generating the integral expression  $F$   
2           comprises:

3           forming a first variable  $\rho = s_{ij}/(\sigma_i\sigma_j)$ ;  
4           forming a second variable  $c = \sqrt{2(1-\rho^2)}$ ;  
5           forming a third variable  $P = (a_i + a_j)/(c\sqrt{2})$ ,  $P$  being one of the integral limits;  
6           forming a fourth variable  $Q = (a_j - a_i)/(c\sqrt{2})$ ; and  
7           forming the integral expression

$$8 \quad F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_P^{\infty} e^{-p^2(1-\rho)} (erf \sqrt{1+\rho}(Q+p+P) - erf(\sqrt{1+\rho}(Q-p+P))) dp$$

9           wherein  $p$  is an integral variable,  $erf$  is an error function,  $a_i$  and  $a_j$  are respectively  
10          equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the vector indices  
11          equal respectively to the row index  $i$  and the column index  $j$ ,  $\mu_i$  and  $\mu_j$  being the means at  
12          the vector indices equal respectively to the row index  $i$  and the column index  $j$ ,  $\sigma_i$  and  $\sigma_j$   
13          being the variances at the vector indices equal respectively to the row index  $i$  and the  
14          column index  $j$ .

- 1           8.     The method of claim 7 wherein obtaining the first non-diagonal element  
2 comprises:  
3           determining a right hand side (RHS) quantity  $g_{ij} = \Sigma_{ij} + \mu_i \mu_j$ ;  
4           equating the integral expression to the RHS quantity to form an integral equation  $F$   
5  $= g_{ij}$ ; and  
6           solving the integral equation for the first variable  $\rho$ .
- 1           9.     The method of claim 8 wherein solving the integral equation comprises:  
2 solving the integral equation using a numerical method.
- 1           10.    The method of claim 6 wherein the first value is 1 and the second value is 0.
- 1           11.    A computer program product comprising:  
2 a machine useable medium having program code embedded therein, the program  
3 code comprising:  
4           computer readable program code to generate a first covariance matrix from  
5 a desired mean vector and a desired covariance matrix of a Bernoulli distribution;  
6           computer readable program code to construct a normal vector using the  
7 desired mean vector and the first covariance matrix; and  
8           computer readable program code to generate a sampling vector using the  
9 normal vector and a threshold vector, the sampling vector having the desired mean  
10 vector and the desired covariance matrix.
- 1           12.    The computer program product of claim 11 wherein the computer readable  
2 program code to generate the first covariance matrix comprises:  
3           computer readable program code to generate an integral expression  $F$  for a first  
4 non-diagonal element  $s_{ij}$  of the first covariance matrix at a row index  $i$  and a column index  
5  $j$ , the integral expression having an integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$   
6 in the threshold vector at the vector indices  $i$  and  $j$ ; and

7 computer readable program code to obtain the first non-diagonal element  $s_{ij}$  using  
 8 the integral expression  $F$ , a mean  $\mu_k$  of the desired mean vector, and a desired non-diagonal  
 9 element  $\Sigma_{ij}$  of the desired covariance matrix.

1 13. The computer program product of claim 12 further comprising:  
 2 computer readable program code to obtain a diagonal element  $s_{jj}$  of the first  
 3 covariance matrix at a first row index  $j$  and a first column index  $j$  using the mean  $\mu_j$  at the  
 4 vector index  $j$ , the diagonal element being equal to a desired diagonal element  $\Sigma_{jj}$  of the  
 5 desired covariance matrix.

1 14. The computer program product of claim 13 further comprising:  
 2 computer readable program code to generate a threshold element  $\tau_j$  of the threshold  
 3 vector at a vector index  $j$ , the threshold element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1-2\mu_j)$   
 4 wherein  $\mu_j$  and  $\sigma_j$  are desired mean and variance, respectively, at the vector index  $j$  and  
 5  $\operatorname{inverf}$  is an inverse error function.

1 15. The computer program product of claim 12 wherein the computer readable  
 2 program code to construct the normal vector comprises:  
 3 computer readable program code to generate normal elements of the normal vector  
 4 using the desired mean vector and the first covariance matrix.

1 16. The computer program product of claim 15 wherein the computer readable  
 2 program code to generate the sampling vector comprises:  
 3 computer readable program code to compare a normal element  $Y_k$  of the normal  
 4 vector at a vector index  $k$  with a corresponding threshold element  $\tau_k$  of the threshold vector  
 5 at the vector index  $k$ ;  
 6 computer readable program code to set a sampling element of the sampling vector  
 7 at the vector index  $k$  to a first value if the normal element  $Y_k$  is greater than the  
 8 corresponding threshold element  $\tau_k$ ; and  
 9 computer readable program code to set the sampling element of the sampling vector  
 10 at the vector index  $k$  to a second value if the normal element  $Y_k$  is equal to or less than the  
 11 corresponding threshold element  $\tau_k$ .

17. The computer program product of claim 12 wherein the computer readable program code to generate the integral expression F comprises:

computer readable program code to form a first variable  $\rho = s_{ij}/(\sigma_i\sigma_j)$ ;

computer readable program code to form a second variable  $c = \sqrt{2(1-\rho^2)}$ ;

computer readable program code to form a third variable  $P = (a_i + a_j)/(c\sqrt{2})$ , P

being one of the integral limits;

computer readable program code to form a fourth variable  $Q = (a_j - a_i)/(c\sqrt{2})$ ; and

computer readable program code to form the integral expression

$$F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_P^{\infty} e^{-p^2(1-\rho)} (\operatorname{erf} \sqrt{1+\rho}(Q+p+P) - \operatorname{erf}(\sqrt{1+\rho}(Q-p+P))) dp$$

wherein p is an integral variable, erf is an error function,  $a_i$  and  $a_j$  are respectively equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the vector indices equal respectively to the row index i and the column index j,  $\mu_i$  and  $\mu_j$  being the means at the vector indices equal respectively to the row index i and the column index j,  $\sigma_i$  and  $\sigma_j$  being the variances at the vector indices equal respectively to the row index i and the column index j.

18. The computer program product of claim 17 wherein the computer readable program code to obtain the first non-diagonal element comprises:

computer readable program code to determine a right hand side (RHS) quantity  $g_{ij} = \Sigma_{ij} + \mu_i\mu_j$ ;

computer readable program code to equate the integral expression to the RHS quantity to form an integral equation  $F = g_{ij}$ ; and

computer readable program code to solve the integral equation for the first variable  $\rho$ .

19. The computer program product of claim 18 wherein the computer readable program code to solve the integral equation comprises:

computer readable program code to solve the integral equation using a numerical method.

1           20.    The method of claim 16 wherein the first value is 1 and the second value is  
2    0.

1           21.    A simulator comprises:  
2           a network modeler to model a network of free-space optical links;  
3           a reliability modeler coupled to the network modeler to evaluate a reliability model  
4    for the network; and  
5           a random sampler coupled to the network modeler and the reliability modeler to  
6    generate random samples for a Bernoulli distribution, the random sampler comprising:  
7           a covariance generator to generate a first covariance matrix from a desired  
8           mean vector and a desired covariance matrix of the Bernoulli distribution,  
9           a normal vector generator coupled to the covariance generator to construct a  
10          normal vector using the desired mean vector and the first covariance matrix, and  
11          a thresholder coupled to the covariance generator and the normal vector  
12          generator to generate a sampling vector using the normal vector and a threshold  
13          vector, the sampling vector having the desired mean vector and the desired  
14          covariance matrix.

1           22.    The simulator of claim 21 wherein the covariance generator comprises:  
2           an integral expression generator to generate an integral expression  $F$  for a first non-  
3    diagonal element  $s_{ij}$  of the first covariance matrix at a row index  $i$  and a column index  $j$ , the  
4    integral expression having an integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$  in  
5    the threshold vector at the vector indices  $i$  and  $j$ ; and  
6           a non-diagonal element generator coupled to the integral expression generator to  
7    obtain the first non-diagonal element  $s_{ij}$  using the integral expression  $F$ , a mean  $\mu_k$  of the  
8    desired mean vector, and a desired non-diagonal element  $\Sigma_{ij}$  of the desired covariance  
9    matrix.

1           23.    The simulator of claim 22 wherein the random sampler further comprises:  
2           a diagonal element generator to obtain a diagonal element  $s_{jj}$  of the first covariance  
3    matrix at a first row index  $j$  and a first column index  $j$  using the mean  $\mu_j$  at the vector index

4 j, the diagonal element being equal to a desired diagonal element  $\Sigma_{jj}$  of the desired  
5 covariance matrix.

1 24. The simulator of claim 23 wherein the random sampler further comprises:  
2 a threshold vector calculator coupled to the first normal vector generator to  
3 generate a threshold element  $\tau_j$  of the threshold vector at a vector index j, the threshold  
4 element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1-2\mu_j)$  wherein  $\mu_j$  and  $\sigma_j$  are the desired mean and  
5 variance, respectively, at the vector index j and inverf is an inverse error function.

1 25. The simulator of claim 22 wherein the normal vector generator generates  
2 normal elements of the normal vector using the desired mean vector and the first  
3 covariance matrix.

1 26. The simulator of claim 25 wherein thresholder comprises:  
2 a comparator to compare a normal element  $Y_k$  of the normal vector at a vector  
3 index k with a corresponding threshold element  $\tau_k$  of the threshold vector at the vector  
4 index k; and  
5 a selector coupled to the comparator to set a sampling element of the sampling  
6 vector at the vector index k to a first value if the normal element  $Y_k$  is greater than the  
7 corresponding threshold element  $\tau_k$  and to set the sampling element of the sampling vector  
8 at the vector index k to a second value if the normal element  $Y_k$  is equal to or less than the  
9 corresponding threshold element  $\tau_k$ .

1 27. The simulator of claim 22 wherein the integral expression generator  
2 generates the integral expression

$$3 \quad F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_P^{\infty} e^{-p^2(1-\rho)} (\operatorname{erf} \sqrt{1+\rho}(Q+p+P) - \operatorname{erf}(\sqrt{1+\rho}(Q-p+P))) dp$$

4 wherein:

5  $\rho = s_{ij}/(\sigma_i \sigma_j)$ ,  $c = \sqrt{2(1-\rho^2)}$ ,  $P = (a_i + a_j)/(c\sqrt{2})$ , P being one of the integral limits,  
6  $Q = (a_j - a_i)/(c\sqrt{2})$ , p is an integral variable, erf is an error function,  $a_i$  and  $a_j$  are

7 respectively equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the  
 8 vector indices equal respectively to the row index  $i$  and the column index  $j$ ,  $\mu_i$  and  $\mu_j$  being  
 9 the means at the vector indices equal respectively to the row index  $i$  and the column index  
 10  $j$ ,  $\sigma_i$  and  $\sigma_j$  being the variances at the vector indices equal respectively to the row index  $i$   
 11 and the column index  $j$ .

1 28. The simulator of claim 27 wherein the non-diagonal element generator  
 2 comprises:  
 3 a right hand side (RHS) generator to determines a right hand side (RHS) quantity  $g_{ij}$   
 4  $= \Sigma_{ij} + \mu_i \mu_j$ ;  
 5 an equation solver coupled to the integral expression generator and the RHS  
 6 generator to equate the integral expression to the RHS quantity to form an integral  
 7 equation  $F = g_{ij}$ , and to solve the integral equation for the first variable  $\rho$ .

1 29. The simulator of claim 28 wherein the equation solver solves the integral  
 2 equation using a numerical method.

1 30. The simulator of claim 26 wherein the first value is 1 and the second value  
 2 is 0.

1 31. A system comprises:  
 2 a processor; and  
 3 a memory coupled to the processor, the memory having program code, the program  
 4 code when executed by the processor causing the processor to:  
 5 generate a first covariance matrix from a desired mean vector and a desired  
 6 covariance matrix of a Bernoulli distribution,  
 7 construct a normal vector using the desired mean vector and the first  
 8 covariance matrix, and  
 9 generate a sampling vector using the normal vector and a threshold vector,  
 10 the sampling vector having the desired mean vector and the desired covariance  
 11 matrix.



1           32.    The system of claim 31 wherein the program code causing the processor to  
 2   generate the first covariance matrix causes the processor to:  
 3           generate an integral expression F for a first non-diagonal element  $s_{ij}$  of the first  
 4   covariance matrix at a row index i and a column index j, the integral expression having an  
 5   integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$  in the threshold vector at the vector  
 6   indices i and j; and  
 7           obtain the first non-diagonal element  $s_{ij}$  using the integral expression F, a mean  $\mu_k$   
 8   of the desired mean vector, and a desired non-diagonal element  $\Sigma_{ij}$  of the desired  
 9   covariance matrix.

1           33.    The system of claim 32 wherein the program code, when executed, further  
 2   causing the processor to:  
 3           obtain a diagonal element  $s_{jj}$  of the first covariance matrix at a first row index j and  
 4   a first column index j using the mean  $\mu_j$  at the vector index j, the diagonal element being  
 5   equal to a desired diagonal element  $\Sigma_{jj}$  of the desired covariance matrix.

1           34.    The system of claim 33 wherein the program code, when executed, further  
 2   causing the processor to:  
 3           generate a threshold element  $\tau_j$  of the threshold vector at a vector index j, the  
 4   threshold element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1-2\mu_j)$  wherein  $\mu_j$  and  $\sigma_j$  are desired  
 5   mean and variance, respectively, at the vector index j and inverf is an inverse error  
 6   function.

1           35.    The system of claim 32 wherein the program code causing the processor to  
 2   construct the normal vector causes the processor to:  
 3           generate normal elements of the normal vector using the desired mean vector and  
 4   the first covariance matrix.

1           36.    The system of claim 35 wherein the program code causing the processor to  
 2   generate the sampling vector causes the processor to:

3 compare a normal element  $Y_k$  of the normal vector at a vector index  $k$  with a  
 4 corresponding threshold element  $\tau_k$  of the threshold vector at the vector index  $k$ ;  
 5 set a sampling element of the sampling vector at the vector index  $k$  to a first value  
 6 if the normal element  $Y_k$  is greater than the corresponding threshold element  $\tau_k$ ; and  
 7 set the sampling element of the sampling vector at the vector index  $k$  to a second  
 8 value if the normal element  $Y_k$  is equal to or less than the corresponding threshold element  
 9  $\tau_k$ .

1 37. The system of claim 32 wherein the program code causing the processor to  
 2 generate the integral expression  $F$  causes the processor to:

3 form a first variable  $\rho = s_{ij}/(\sigma_i\sigma_j)$ ;

4 form a second variable  $c = \sqrt{2(1-\rho^2)}$ ;

5 form a third variable  $P = (a_i + a_j)/(c\sqrt{2})$ ,  $P$  being one of the integral limits;

6 form a fourth variable  $Q = (a_j - a_i)/(c\sqrt{2})$ ; and

7 form the integral expression

$$8 \quad F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_P^\infty e^{-\rho^2(1-\rho)} (erf(\sqrt{1+\rho}(Q+p+P)) - erf(\sqrt{1+\rho}(Q-p+P))) dp$$

9 wherein  $p$  is an integral variable,  $erf$  is an error function,  $a_i$  and  $a_j$  are respectively  
 10 equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the vector indices  
 11 equal respectively to the row index  $i$  and the column index  $j$ ,  $\mu_i$  and  $\mu_j$  being the means at  
 12 the vector indices equal respectively to the row index  $i$  and the column index  $j$ ,  $\sigma_i$  and  $\sigma_j$   
 13 being the variances at the vector indices equal respectively to the row index  $i$  and the  
 14 column index  $j$ .

1 38. The system of claim 32 wherein the program code causing the processor to  
 2 obtain the first non-diagonal element causes the processor to:

3 determine a right hand side (RHS) quantity  $g_{ij} = \Sigma_{ij} + \mu_i\mu_j$ ;

4 equate the integral expression to the RHS quantity to form an integral equation  $F =$   
 5  $g_{ij}$ ; and

6 solve the integral equation for the first variable  $\rho$ .

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1            39.    The system of claim 38 wherein the program code causing the processor to  
2    solve the integral equation causes the processor to:  
3            solve the integral equation using a numerical method.

1            40.    The system of claim 36 wherein the first value is 1 and the second value is  
2    0.